Recall: The simplex method works under the following assumptions:

1) The program is in the equality form: we want to maximize

$$z = c_1x_1 + \ldots + c_nx_n$$

subject to the constraints

$$\begin{array}{c} a_{11}x_1 + \ldots + a_{1n}x_n = b_1 \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_{m1}x_1 + \ldots + a_{mn}x_n = b_m \end{array}$$

$$x_1,\ x_2,\ \ldots, x_n \geq 0$$

2) The coefficient matrix

$$A = \begin{bmatrix} a_{11} & \ldots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \ldots & a_{mn} \end{bmatrix}$$

is in the basic form.

3) $b_i \geq 0$ for $i = 1, \ldots, m$.

- These assumptions assure that the program has a basic feasible solution.

- **Phase I of the simplex method** finds some basic feasible solution or verifies that no feasible solutions exists.

Example. Maximize

$$z = x_1 + 2x_2$$

subject to:

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 2 \\ -x_2 - x_3 &= -1 \end{aligned}$$

$$x_1,\ x_2, x_3 \geq 0$$

Phase I of the simplex method

Assumption: The linear program is in the equality form with constraints

$$a_{i1}x_1 + \ldots + a_{in}x_n = b_i$$

for $i = 1, \ldots, m$.

1) Modify the constraints, if needed, so that $b_i \geq 0$ for $i = 1, \ldots, m$.

2) Add an additional variable s_i to the i-th constraint for $i = 1, \ldots, m$.

Note. The augmented matrix of the new constraints is in the basic form.

3) Use the simplex method to minimize the function

$$z = s_1 + \cdots + s_m$$

with the new constraints. If the minimum is non-zero, then the original linear program has not feasible solutions.

If the minimum is $z = 0$, then the solution that gives the minimum has $s_i = 0$ for $i = 1, \ldots, m$. In such case, values of the variables $x_1, \ldots, x_n$ give a basic feasible solution of the original linear program.