The *simplex method* is one of the main methods of solving linear programs.

**Special assumptions (for now):**

**1)** The program is in the equality form: we want to maximize

$$z = c_1x_1 + \ldots + c_nx_n$$

subject to the constraints:

$$\begin{array}{c} a_{11}x_1 + \ldots + a_{1n}x_n = b_1 \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_{m1}x_1 + \ldots + a_{mn}x_n = b_m \end{array}$$

$$x_1,\ x_2,\ \ldots, x_n \geq 0$$

**2)** The coefficient matrix

$$A = \begin{bmatrix} a_{11} & \ldots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \ldots & a_{mn} \end{bmatrix}$$

is in the basic form.

**3)** $b_i \geq 0$ for $i = 1, \ldots, m$.

**Example.**

$$\left[\begin{array}{rrrrr|r} -1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 3 \\ 2 & 0 & 0 & 1 & 1 & 7 \end{array}\right]$$

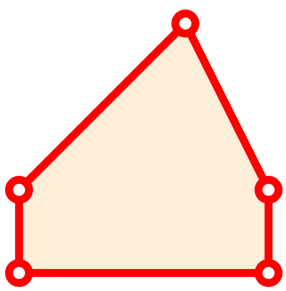
## Basic feasible solutions

*Basic feasible solutions* are the solutions obtained by setting all free variables to 0.

**Example.**

$$\left[\begin{array}{rrrrr|r} -1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 3 \\ 2 & 0 & 0 & 1 & 1 & 7 \end{array}\right] \qquad \begin{cases} x_3 = 1 + x_1 - x_4 \\ x_2 = 3 - x_1 \\ x_5 = 7 - 2x_1 - x_4 \\ x_1 = \text{free} \\ x_4 = \text{free} \\ x_1, x_2, x_3, x_4, x_5 \geq 0 \end{cases}$$

## Geometric interpretation



## The pivot step

$$\left[\begin{array}{rrrrr|r} -1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 3 \\ 2 & 0 & 0 & 1 & 1 & 7 \end{array}\right]$$

**One more assumption**

**4)** In the objective function $z = c_1x_1 + \ldots + c_nx_n$ the coefficients $c_i$ corresponding to basic variables are equal to 0.

**Example.** The objective function:

$$4x_1 + 2x_2 + x_4 = z$$

Constraints:

$$\begin{aligned} -x_1 + x_3 + x_4 &= 1 \\ x_1 + x_2 &= 3 \\ 2x_1 + x_4 + x_5 &= 7 \end{aligned}$$

$$x_1,\ x_2,\ x_3,\ x_4,\ x_5 \geq 0$$