Assume that we have an absorbing Markov chain with

- absorbing states $S_1, \ldots, S_M$
- non-absorbing states $S_{M+1}, \ldots, S_N$
- the transition matrix

$$P = \begin{array}{cc} \text{abs.} & \text{non-abs.} \\ \left[\begin{array}{c} I \\ 0 \end{array}\right. & \left.\begin{array}{c} S \\ Q \end{array}\right] \end{array} \begin{array}{l} \text{abs.} \\ \text{non-abs.} \end{array}$$

Questions:

1) If the chain starts in an non-absorbing state S_i, how many steps it will take on the average before it transitions to an absorbing state?

2) If the chain starts in an non-absorbing state S_i, how many times, on the average, it will visit a non-absorbing state S_j before being absorbed?

Random variables and expected values

Example. In a certain game a player can:

- loose \$2 with the probability 0.5
- win \$2 with the probability 0.3
- win \$10 with the probability 0.2

How much will the player win per game on the average?

Definition

A *discrete random variable* X consists of

- A set of values (outcomes) $v_i \in \mathbb{R}$ for $i = 1, 2, \ldots$.
- Probabilities p_i that X assumes each of the values v_i:

$$P(X = v_i) = p_i$$

We have $0 \leq p_i \leq 1$ and $\sum p_i = 1$.

Definition

If X is a discrete random variable with values $v_i \in \mathbb{R}$ then the *expected value* of X is the number

$$E[X] = \sum_i v_i P(X = v_i)$$

Proposition

If $X_1, \dots, X_m$ are discrete random variables with values in $\mathbb{R}$ then

$$E[X_1 + \dots + X_m] = E[X_1] + \dots + E[X_m]$$

Back to absorbing Markov chains

Recall: We have an absorbing Markov chain with

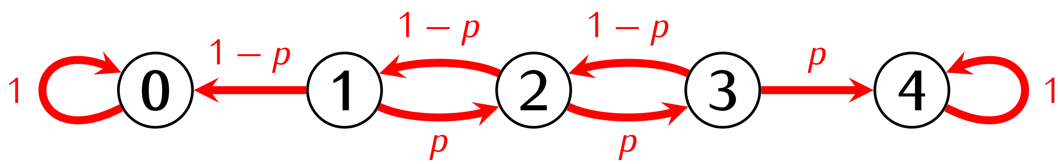
- absorbing states $S_1, \ldots, S_M$
- non-absorbing states $S_{M+1}, \ldots, S_N$
- the transition matrix

$$P = \begin{array}{cc} \begin{array}{cc} \text{abs.} & \text{non-abs.} \end{array} & \\ \begin{bmatrix} I & S \\ 0 & Q \end{bmatrix} & \begin{array}{l} \text{abs.} \\ \text{non-abs.} \end{array} \end{array}$$

Question. If the chain starts in an non-absorbing state S_i, how many times, on the average, it will visit a non-absorbing state S_j before being absorbed?

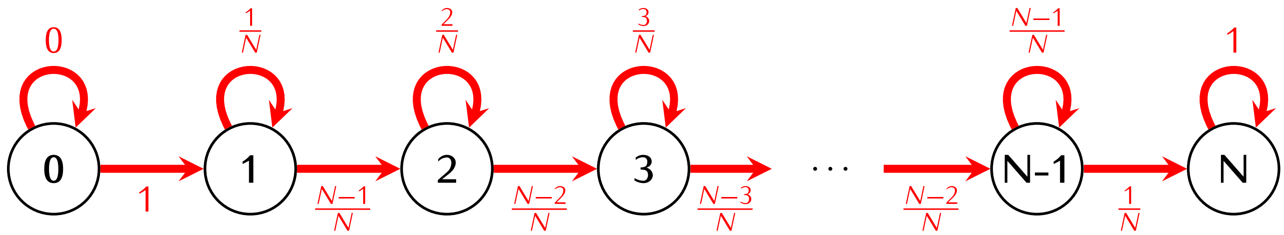
Simplification.

Example. The gambling model (with $p \neq 0, 1$):



$$P = \begin{array}{c} \\ 0 \\ 4 \\ 1 \\ 2 \\ 3 \end{array} \!\!\! \begin{array}{c} \begin{array}{ccccc} \;\;0\;\; & \;\;4\;\; & \;\;1\;\; & \;\;2\;\; & \;\;3\;\; \end{array} \\ \left[\begin{array}{ccccc} 1 & 0 & 1-p & 0 & 0 \\ 0 & 1 & 0 & 0 & p \\ 0 & 0 & 0 & 1-p & 0 \\ 0 & 0 & p & 0 & 1-p \\ 0 & 0 & 0 & p & 0 \end{array}\right] \end{array}$$

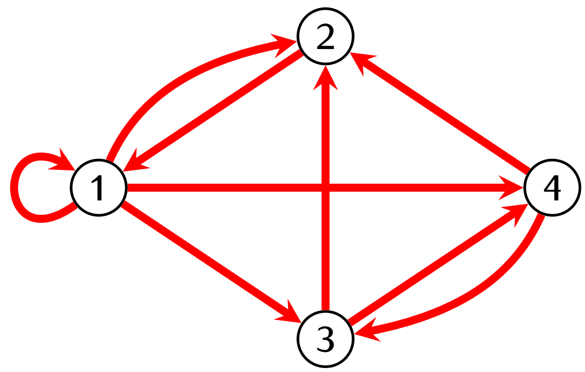
Example. Toy collecting:



Question. How many steps, on the average it will take to collect all toys?

Transit time

Example. Consider a random walk on the following directed network:



How many steps will it take on the average to get from node 2 to node 3?