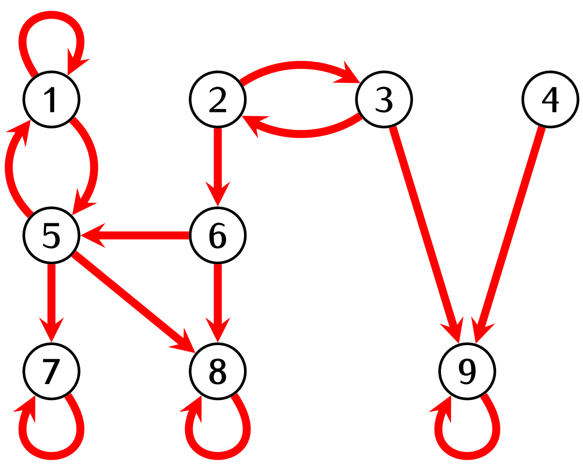
Definition

Consider a Markov chain with states $S_1, \ldots, S_N$ and the transition matrix $P = (p_{ij})$.

- A state S_i is *absorbing* if $p_{ii} = 1$
- The Markov chain is *absorbing* if for each state there is a non-zero probability that the state will transition to an absorbing state after some number of steps.

Example.



Transition matrix of an absorbing Markov chain

Proposition

Consider an absorbing Markov chain with the transition matrix in the form

$$P = \begin{array}{cc} \begin{array}{cc} \text{abs.} & \text{non-abs.} \end{array} & \\ \begin{bmatrix} I & S \\ 0 & Q \end{bmatrix} & \begin{array}{l} \text{abs.} \\ \text{non-abs.} \end{array} \end{array}$$

Then the following hold:

- $\lim_n Q^n = 0$
- The infinite series $I + Q + Q^2 + \dots$ converges.
- $I + Q + Q^2 + \cdots = (I - Q)^{-1}$

Definition

For an absorbing Markov chain the matrix

$$(I - Q)^{-1} = I + Q + Q^2 + \dots$$

is called the *fundamental matrix* of the Markov chain.

Corollary

For an absorbing Markov chain the transition matrix

$$P = \begin{array}{cl} \begin{array}{cc} \text{abs.} & \text{non-abs.} \end{array} & \\ \begin{bmatrix} I & S \\ 0 & Q \end{bmatrix} & \begin{array}{l} \text{abs.} \\ \text{non-abs.} \end{array} \end{array}$$

we have:

$$\lim_n P^n = \begin{bmatrix} I & S(I - Q)^{-1} \\ 0 & 0 \end{bmatrix}$$

Example. The gambling model: