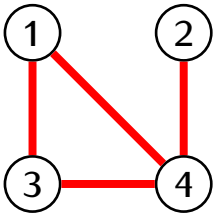
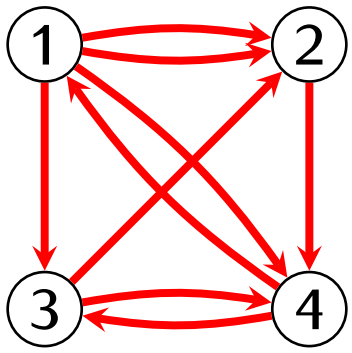
Steady state vector of a random walk on an undirected connected network

Example.



Random walks on directed networks and the Google PageRank

Example. Network of web pages:



Question. How to rank web pages?

PageRank:

- Consider the steady-state vector of a random walk on the network of pages. It gives probabilities that a walker will visits each page in a long run.
- Higher probability of a page means that the page is more popular.
- Rank pages according to these probabilities.

Recall:

Definition

A stochastic matrix P is *regular* if there is $N \geq 0$ such that all entries of P^N are positive.

Perron-Frobenius Theorem

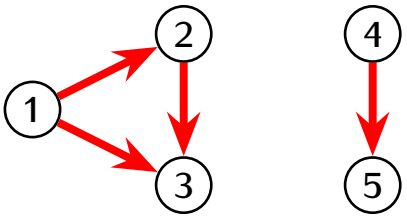
If P is a regular stochastic matrix then:

- There exists only one steady state vector Y of P
- For any probability vector X we have

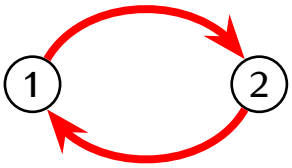
$$\lim_n P^n X = Y$$

Note. The transition matrix for a random walk on a network of web pages need not be regular.

Problem: a disconnected network.



Problem: cycles.



Solution.