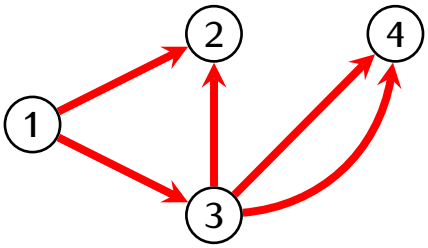
Definition

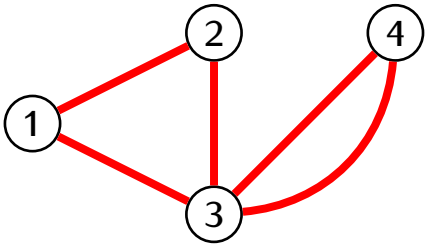
For a graph with vertices $1, 2, 3, \ldots, N$ the *adjacency matrix* of the graph is an $N \times N$ matrix $A = (a_{ij})$ such that

$$a_{ij} = (\text{the number of edges from } j \text{ to } i)$$

Example. Directed graph:

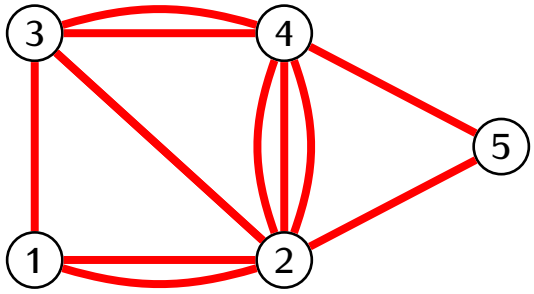


Example. Undirected graph:



Definition

A *path* in a graph is a sequence of edges such that each edge ends at the vertex when the next edge begins.



Example. In the graph pictured above, how many paths of length 2 are there that start at the vertex 2 and end at the vertex 4?

Proposition

Let A be the the adjacency matrix of a graph.

The entry b_{ij} of the matrix $A^2 = (b_{ij})$ gives the number of paths of length 2 that start at the vertex j and terminate at the vertex i.

In general, for any $n \geq 1$ the entry c_{ij} of the matrix $A^n = (c_{ij})$ gives the number of paths of length n that start at the vertex j and terminate at the vertex i.