Example. A company needs to hire people for 5 different positions $P_1, \ldots, P_5$. There are 7 candidates $C_1, \ldots, C_7$ who interviewed for these positions. The table below shows the interview score (higher is better) how each person is qualified for each position. Blank entries indicate the score of 0 (i.e. a candidate is either not suitable or not interested in the corresponding position).

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
P_1	70	90		75	55		60
P_2	40	95	85			80	
P_3	50		75		70		65
P_4			60	80		35	
P_5		75		70		35	20

Which candidate should be offered which position so that the sum of scores of the assignment is the largest possible?

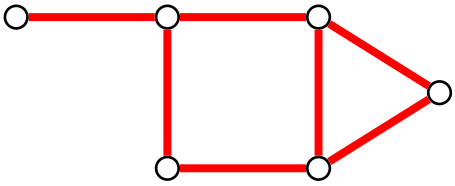
Goal: All basic feasible solutions of the assignment problem consist of integers.

Definition

A *graph* (or a *network*) is a pair $G = (V, E)$ where:

- V is the set of *vertices* (or *nodes*);
- E is the set of *edges*;
- each edge connects two vertices.

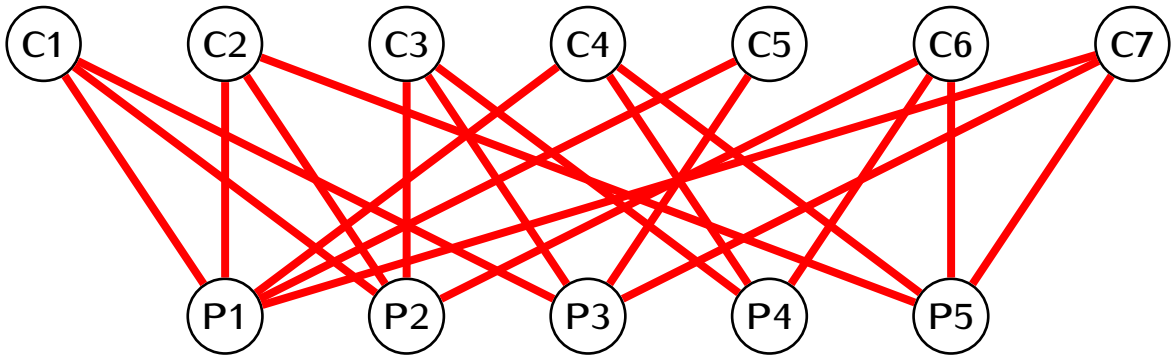
Example.



Definition

A *bipartite graph* is a graph $G = (V, E)$ such the set of nodes is a union of two disjoint subsets $V = V_1 \cup V_2$ and that every edge connects some node in V_1 with some node in V_2.

Example. Bipartite graph for the assignment problem:

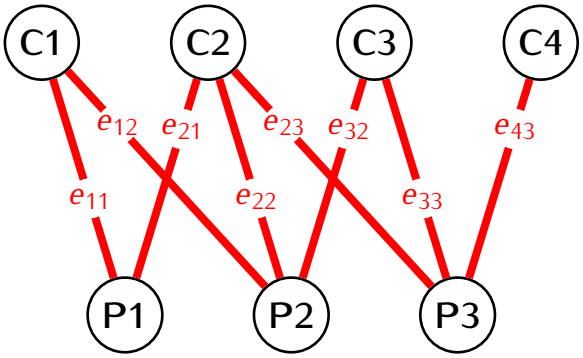


Definition

The *edge incidence matrix* of a graph $G = (V, E)$ is a matrix A such that:

- rows of A are labeled by vertices of G
- columns of A are labeled by edges of G
- the entry in the row of a vertex **v** and the column of an edge **e** is 1 if the edge **e** is attached to **v**; otherwise it is 0.

Example.



	e_{11}	e_{12}	e_{21}	e_{22}	e_{23}	e_{32}	e_{33}	e_{43}
C1	1	1	0	0	0	0	0	0
C2	0	0	1	1	1	0	0	0
C3	0	0	0	0	0	1	1	0
C4	0	0	0	0	0	0	0	1
P1	1	0	1	0	0	0	0	0
P1	0	1	0	1	0	1	0	0
P3	0	0	0	0	1	0	1	1