Note.

Exercises 5-7 involve the simplex method. You already know enough about the simplex method to solve these problems. Alternatively, since this homework is due in two weeks, you can wait which these problems until Monday (2/13) when I will finish the simplex method example.

1. A company manufacturing widgets has the following demand for widgets for the first four months of the year:

	January	February	March	April
widgets ordered	500	1000	1600	1400

The company has to decide how many widgets it should produce each month.

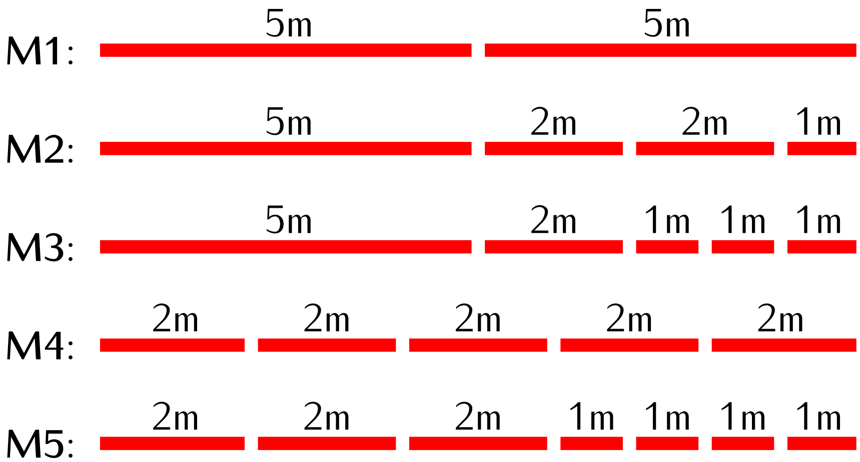
It costs \$5 to produce each widget. The maximum production per month is 1200 widgets. Widgets produced one month can be eithr delivered to customers that same month or stored in a warehouse and delivered some other month. It costs the company \$3 to store one widget from one month to the next.

a) Formulate as a linear program the problem of determining how many widgets should be produced each month to meet the demand and minimize the total production and storage costs.

b) Assume that by working overtime the company can manufacture up to 400 additional widgets per month but it costs \$10 to produce each of these additional widgets. Formulate a linear program that minimized the total production and storage costs in this case.

In each case explain what are the decision variables, the objective function, and the constraints.

2. A steel company manufactures 10 meter long metal rods. The rods are then cut into smaller pieces that are sold to customers. The company has five machines **M1** – **M5** that cut the rods as follows.



The company received an order for

- 175 rods of length 5m
- 100 rods of length 2m
- 225 rods of length 1m.

The company needs to plan how many 10 meter rods should be cut using each machine to fulfill this order.

Formulate a linear program that will minimize the total number of 10 meter rods that need to be cut. Explain what are the decision variables, the objective function, and the constraints.

3. Convert the following linear programming problem to the equality form:

Minimize

$$z = x_1 - x_2$$

subject to constraints

$$\begin{aligned} 2x_1 + x_2 &\geq 3 \\ 3x_1 - x_2 &\leq 7 \\ x_1 &\geq 0 \\ x_2 &\in \mathbb{R} \end{aligned}$$

4. For the following linear programming problem, plot the feasible region and use it to solve the problem (no simplex method required, just use the plot).

Find the maximum of the function

$$z = 5x_1 + 7x_2$$

subject to constraints

$$\begin{aligned} 2x_1 + 3x_2 &\geq 6 \\ 3x_1 - x_2 &\leq 15 \\ -x_1 + x_2 &\leq 4 \\ 2x_1 + 5x_2 &\leq 27 \\ x_1 &\geq 0 \\ x_2 &\geq 0 \end{aligned}$$

5. Solve the following linear program using the simplex method. At each pivot step write the basic feasible solution obtained and indicate which variable becomes basic and which becomes free.

a) Maximize the function

$$z = 4x_1 + 6x_2$$

subject to constraints:

$$\begin{aligned} -x_1 + x_2 &\leq 11 \\ x_1 + x_2 &\leq 27 \\ 2x_1 + 5x_2 &\leq 90 \\ x_1, x_2 &\geq 0 \end{aligned}$$

b) Sketch the feasible region for above linear program, and number the extreme points in the order they appear in your simplex method calculations.

6. Assume that in the process of solving a linear program using the simplex method, you obtain the following tableau:

x_1	x_2	$\dots\dots\dots$	
a_1	1	$\dots\dots\dots$	b_1
a_2	0	$\dots\dots\dots$	b_2
$\dots$	$\dots$	$\dots\dots\dots$	$\dots$
a_n	0	$\dots\dots\dots$	b_n
c_1	0	$\dots\dots\dots$	$z - d$

Here $c_1 \neq 0$. The variable x_1 is free and x_2 is basic. Assume also that in the next pivot step x_1 becomes basic and x_2 free. Explain why x_2 cannot become basic again in the pivot step that occurs immediately after that.

Hint. What can you say about c_1 and a_1 in this tableau? Why?

7. Assume that the simplex tableau of a linear program has n variables and m constraints (with $n \geq m$). What is the largest number of basic feasible solutions this linear program can have? Why?

Note. In the worst case, the simplex method may travel through most of these basic feasible solutions before it reaches the maximum, but typically it gets to the maximum much faster.