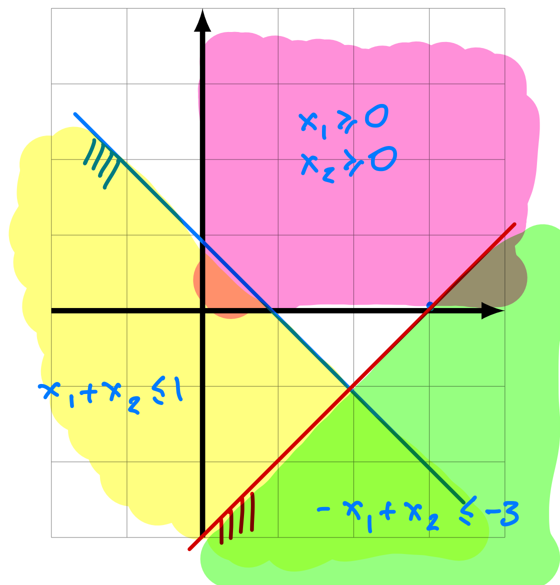
Cases when a solution of a linear program may not exist:

Infeasibilty: There are no feasible solutions.

Example. Maximize $z = 2x_1 + x_2$ subject to

$$x_1 + x_2 \leq 1$$
$$-x_1 + x_2 \leq -3$$
$$x_1,\ x_2 \geq 0$$



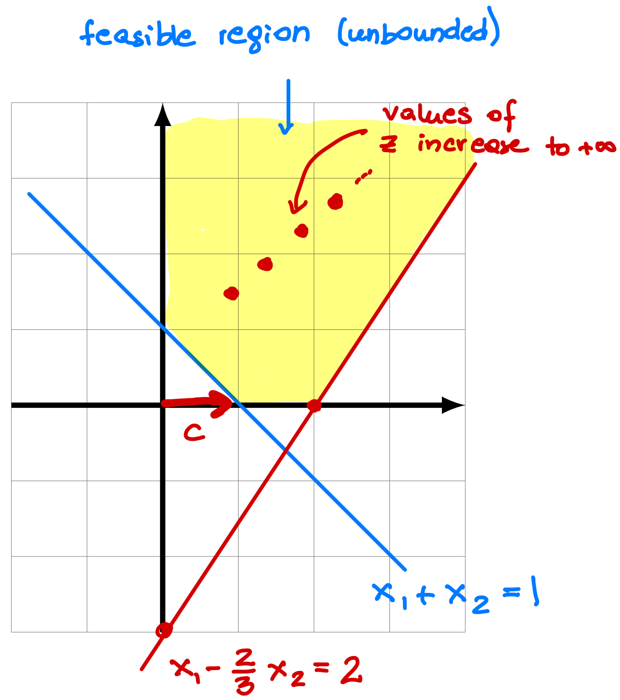
no feasible solutions

Unboundedness: The objective function has no minimum (or maximum) in the feasible region.

$$\overset{c}{\begin{bmatrix}1\\0\end{bmatrix}} \cdot \overset{x}{\begin{bmatrix}x_1\\x_2\end{bmatrix}} = z$$

Example. Maximize $z = x_1 + 0x_2$ subject to

$$\begin{aligned} x_1 - \tfrac{2}{3}x_2 &\leq 2 \\ x_1 + x_2 &\geq 1 \\ x_1,\ x_2 &\geq 0 \end{aligned}$$



Note. Even when the feasible region is unbounded the objective function may have a maximum or a minimum in this region.